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www.elsevier.com/locate/physletbNB BLG model in $N = 8$ superfieldsIgor A. Bandos^{a,b,*}^a Department of Theoretical Physics and History of Science, The Basque Country University (EHU/UPV), PO Box 644, 48080 Bilbao, Spain^b Institute for Theoretical Physics, NSC Kharkov Institute of Physics & Technology, UA 61108 Kharkov, Ukraine

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ABSTRACT

We develop the $N = 8$ superfield description of the Bagger–Lambert–Gustavsson (BLG) model in its Nambu bracket (NB) realization. The basic ingredient is the octet of scalar $d = 3$, $N = 8$ superfields ϕ^I depending also on the coordinates of a compact three-dimensional space M_3 . It is restricted by the superembedding-like equation, $\mathbb{D}_{\dot{A}}\phi^I = 2i\psi_B\gamma^I_{BA}$, which can be treated as covariantization of the linearized superembedding equation for supermembrane (M2-brane) with respect to volume preserving diffeomorphisms of M_3 . The curvatures of SDiff₃ connection are expressed through ϕ^I by the $N = 8$ superfield generalization of the BLG Chern–Simons equation (super-CS equation). We show how the dynamical BLG equations appear when studying consistency of these basic equations.

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1. Introduction

Recently, motivated by the search for the Lagrangian of multiple M2-brane system, Bagger, Lambert [1] and Gustavsson [2] proposed the $d = 3$, $N = 8$ supersymmetric action based on Filippov 3-algebra instead of Lie algebra. A particular infinite dimensional 3-algebra related with three-dimensional volume preserving diffeomorphism group SDiff₃ is given in terms of Nambu brackets (Nambu–Poisson brackets) [3] on a three-dimensional compact manifold M_3 . For three functions on M_3 , $\Phi(y^i)$, $\Xi(y^i)$, $\Omega(y^i)$ ($i = 1, 2, 3$), the Nambu brackets are defined by

$$\{\Phi, \Xi, \Omega\} := \bar{e}^{-1}\epsilon^{ijk}\partial_i\Phi\partial_j\Xi\partial_k\Omega, \quad \partial_i := \frac{\partial}{\partial y^i} \quad (1.1)$$

(see [4,5] and references therein; here, following [6,7], we have introduced a fixed M_3 density $\bar{e} = \bar{e}(y)$ in the definition of Nambu brackets).

As it was stressed in [4], Bagger–Lambert–Gustavsson model with Nambu bracket realization of the 3-algebra (NB BLG model) can be treated as 6-dimensional field theory. The BLG gauge fields become the gauge fields for the 3-volume preserving diffeomorphisms (see [8] as well as [7] and references therein). The SDiff₃ gauge potential is given by the 1-form $s^i = dx^\mu s^i_\mu$ on \mathbb{R}^{1+2} obeying the conditions $\partial_i(\bar{e}s^i) = 0$. The SDiff₃ field strength $F^i := ds^i + s^j \wedge \partial_j s^i$ is also M_3 divergenceless, $\partial_i(\bar{e}F^i) = 0$,

$$F^i := ds^i + s^j \wedge \partial_j s^i, \quad \partial_i(\bar{e}F^i) = 0 \quad \Leftarrow \quad \partial_i(\bar{e}s^i) = 0 \quad (1.2)$$

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(see [7] for more details).

Furthermore, the authors of [4] proposed the identification of the NB BLG model with M-theory 5-brane (M5-brane, see [9] for equations of motion and [10] for the covariant action) with the worldvolume chosen to be $\mathbb{R}^{1+2} \otimes M_3$. However, an attempt to obtain the NB BLG model from light-cone M5-brane [6] has resulted only in reproducing the Carrollian limit of the NB BLG model. This suggests to study the NB realization of BLG model separately, and this was the subject of [5–7,11] and also of the original papers [4].

In this Letter we present $N = 8$ superfield description of the NB BLG model. It is the on-shell superfield description which does not allow for constructing the action, but reproduce equations of motion as the selfconsistency conditions of the basic equations.

2. Basic superfield equations

2.1. Superembedding-like equation for octet of $d = 3$, $N = 8$ scalar superfields

The complete on-shell $N = 8$ superfield description of the Nambu bracket realization of the Bagger–Lambert–Gustavsson model (NB BLG model) is provided by the octet of scalar $d = 3$, $N = 8$ superfields, $\phi^I = \phi^I(x^\mu, \theta^{\dot{\alpha}}; y^i)$, depending on additional coordinates y^i ($i = 1, 2, 3$) of a compact space M_3 , which obeys the following basic equation

$$\mathbb{D}_{\dot{A}}\phi^I = 2i\tilde{\gamma}_{AB}^I\psi_{\alpha B}. \quad (2.1)$$

Here and below $\alpha, \beta, \gamma = 1, 2$ are spinorial and $a, b, c = 0, 1, 2$ are vector indices of SO(1, 2), $\tilde{\gamma}_{AB}^I := \gamma_{BA}^I$ are the SO(8) Klebsh–Gordan coefficients relating $\mathbf{8}_v$, $\mathbf{8}_s$ and $\mathbf{8}_c$ representation, which

obey $\gamma^{(I}\tilde{\gamma}^{J)} = \delta^{IJ}I_5$, $\tilde{\gamma}^{(I}\gamma^{J)} = \delta^{IJ}I_5$,² and $\psi_{\alpha B}$ is a fermionic superfield which is expressed through ϕ^I by the γ^I -trace part of Eq. (2.1). Finally $\mathbb{D}_{\alpha\dot{A}}$ is the covariant Grassmann derivative. It is covariant with respect to $d = 3$, $N = 8$ supersymmetry and under the volume preserving diffeomorphisms of M_3 (SDiff₃ group). Hence it involves a fermionic SDiff₃ connection $\zeta_{\alpha\dot{A}}^i$ and, when act on SDiff₃ scalars (like ϕ^I and $\psi_{\alpha B}$), reads $\mathbb{D}_{\alpha\dot{A}} = D_{\alpha\dot{A}} + \zeta_{\alpha\dot{A}}^i \partial_i$ with $D_{\alpha\dot{A}} = \frac{\partial}{\partial \theta^{\alpha\dot{A}}} + i\gamma_{\alpha\dot{A}}^{\mu} \theta_{\dot{A}}^{\beta} \partial_{\mu}$ ($\partial_{\mu} := \frac{\partial}{\partial x^{\mu}}$). These SDiff₃ covariant derivatives obey³

$$\{\mathbb{D}_{\alpha\dot{A}}, \mathbb{D}_{\beta\dot{B}}\} = 2i\gamma_{\alpha\beta}^{\mu} \delta_{\dot{A}\dot{B}} \mathcal{D}_{\mu} + 2i\epsilon_{\alpha\beta} W_{\dot{A}\dot{B}}^i \partial_i, \quad (2.2)$$

where \mathcal{D}_{μ} is the vector covariant derivative, which reads as $\mathcal{D}_{\mu} = \partial_{\mu} + is_{\mu}^i \partial_i$ when acting on SDiff₃ scalars. It involves the vector SDiff₃ gauge potential s_{μ}^i defined by $s^i = d\theta_{\dot{A}}^{\alpha} \zeta_{\alpha\dot{A}}^i + (dx^{\mu} - id\theta_{\dot{A}}^{\alpha} \gamma^{\mu}_{\alpha\dot{A}}) s_{\mu}^i$. The matrices $\gamma_{\alpha\beta}^{\mu}$ in (2.2) are real and symmetric; they obey $\gamma^{(\mu} \tilde{\gamma}^{\nu)} = \eta^{\mu\nu} \delta_{\alpha\beta}^{\gamma}$, where $\tilde{\gamma}^{\mu\alpha\beta} = \epsilon^{\alpha\gamma} \epsilon^{\beta\delta} \gamma_{\gamma\delta}^{\mu}$, with $\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha} = i\tau^2 = \text{antidiag}(1, -1)$, and $\eta^{\mu\nu} = \text{diag}(1, -1, -1)$, is the flat metric in the $d = 3$ spacetime.

Finally, $W_{\dot{A}\dot{B}}^i$ is the basic superfield strength of the SDiff₃ gauge supermultiplet. This carries the indices of **28** representation of SO(8), i.e., $W_{\dot{A}\dot{B}}^i = -W_{\dot{B}\dot{A}}^i$, and is a vector field with respect to SDiff₃ gauge group.

2.2. $N = 8$ superfield generalization of the Chern–Simons gauge field equation

We impose on $W_{\dot{A}\dot{B}}^i$ the superfield generalization of the Chern–Simons field equation. This reads⁴

$$W_{\dot{A}\dot{B}}^i = \bar{e}^{-1} \epsilon^{ijk} \partial_i \phi^I \partial_j \phi^J \tilde{\gamma}_{\dot{A}\dot{B}}^{IJ}. \quad (2.3)$$

Notice that $W_{\dot{A}\dot{B}}^i$ in (2.3) automatically satisfies the condition $\partial_i (\bar{e} W_{\dot{A}\dot{B}}^i) = 0$ necessary for any SDiff₃ field strength [7] (see (1.2)).

As far as $\tilde{\gamma}_{\dot{A}\dot{B}}^{IJ}$ form the complete basis in the space of anti-symmetric 8×8 matrices, an equivalent form of the super Chern–Simons equation (2.3) is given by

$$W^{IJi} = \bar{e}^{-1} \epsilon^{ijk} \partial_i \phi^I \partial_j \phi^J, \quad W_{\dot{A}\dot{B}}^i =: W^{IJi} \tilde{\gamma}_{\dot{A}\dot{B}}^{IJ}. \quad (2.4)$$

3. Bagger–Lambert equations of motion from the basic superfield equations

The spinor covariant derivative ‘algebra’ (2.2) simulates the constraints for SYM fields. However, Eq. (2.3) implies that the corresponding SDiff₃ gauge theory supermultiplet is composed in the sense that all the field strengths are expressed through the scalar and spinor fields.

Indeed, studying the consequences of the gauge field Bianchi identities⁵ one finds, firstly, that the commutator of vector and spinor covariant derivatives reads $[\mathbb{D}_{\alpha\dot{B}}, \mathcal{D}_a] = i\gamma_{\alpha\beta} W_{\dot{B}}^{\beta i} \partial_i$ and that the Grassmann spinor octet field strength $W_{\alpha\dot{B}}^i$ is given by

$$W_{\alpha\dot{B}}^i = \frac{i}{7} \mathbb{D}_{\alpha\dot{A}} W_{\dot{A}\dot{B}}^i = 4\bar{e}^{-1} \epsilon^{ijk} \partial_j \phi^I \partial_k \psi_{\alpha A} \gamma_{\dot{A}\dot{B}}^J \quad (3.1)$$

and, secondly, that tensorial gauge field strength $([\mathcal{D}_{\mu}, \mathcal{D}_{\nu}] = F_{\mu\nu}^i \partial_i)$ reads⁶ $F_{\mu\nu}^i = -\frac{1}{16} \epsilon_{\mu\nu\rho} \gamma_{\alpha\beta}^{\rho} \mathbb{D}_{\dot{A}}^{\alpha} W_{\dot{A}}^{\beta i}$ so that

$$F_{\mu\nu}^i = -2\bar{e}^{-1} \epsilon^{ijk} \epsilon_{\mu\nu\rho} (\partial_j \phi^I \partial_k \mathcal{D}^{\rho} \phi^J + 2i \partial_j \psi_A \gamma^{\rho} \partial_k \psi_A). \quad (3.2)$$

In (3.2) one recognizes the Chern–Simons type gauge field equations which can be obtained from the BLG Lagrangian of [1]. This expresses the tensorial gauge field strength through the matter (super)fields.

The dynamical bosonic and fermionic equations of motion of the NB BLG model follow from the superembedding-like equation (2.1) and the super-CS equation (2.3). Indeed, with the use of (2.2), one finds that the selfconsistency condition for Eq. (2.1) gives the expression for Grassmann covariant derivative of the fermionic superfield $\psi_{\beta B}$ in (2.1),

$$\begin{aligned} \mathbb{D}_{\alpha\dot{A}} \psi_{\beta B} &= \frac{1}{2} \gamma_{\alpha\beta}^{\mu} \mathcal{D}_{\mu} \phi^I \gamma_{\dot{A}\dot{B}}^I + \frac{1}{3! \bar{e}} \epsilon_{\alpha\beta} W^{IJi} \partial_i \phi^K \tilde{\gamma}_{\dot{A}\dot{B}}^{IJK} \\ &= \frac{1}{2} \mathcal{D}_{\alpha\beta} \phi^I \tilde{\gamma}_{\dot{A}\dot{B}}^I + \frac{1}{6} \epsilon_{\alpha\beta} \{\phi^I, \phi^J, \phi^K\} \tilde{\gamma}_{\dot{A}\dot{B}}^{IJK}. \end{aligned} \quad (3.3)$$

Next stage is to study the selfconsistency conditions for Eq. (3.3). Its SO(1,2) vector and SO(8) tensor ($\propto \tilde{\gamma}^{IJKL} \gamma_{\alpha\beta}^{\mu}$) irreducible part gives us the expression (3.1) for the spinor field strength of the SDiff₃ gauge field (fermionic superpartner of the BLG Chern–Simons equation (3.2)).⁷ Taking this into account in the SO(1,2) vector–SO(8) scalar ($\propto \delta_{\dot{A}\dot{B}} \gamma_{\alpha\beta}^I$) irreducible part we obtain the BLG Dirac equation

$$\gamma_{\alpha\beta}^{\mu} \mathcal{D}_{\mu} \psi_{\beta}^{\beta} = -\frac{1}{\bar{e}} \epsilon^{ijk} \partial_i \phi^I \partial_j \phi^J \partial_k \psi_{\alpha A} \gamma_{\dot{A}\dot{B}}^{IJ}, \quad (3.4)$$

which can be equivalently written in the following compact form

$$\mathcal{D}\psi = -\{\phi^I, \phi^J, \psi\} \gamma^{IJ}. \quad (3.5)$$

As usually, the bosonic equations of motion can be obtained by taking the covariant spinorial derivative of the fermionic ones. Acting by the covariant (SDiff₃ and SUSY covariant) spinor derivatives on (3.4), and extracting the $\propto \epsilon_{\alpha\beta} \gamma_{\dot{A}\dot{B}}^I$ irreducible part one finds

$$\mathcal{D}^{\mu} \mathcal{D}_{\mu} \phi^I = 2\{\phi^J, \phi^K, \{\phi^I, \phi^J, \phi^K\}\} - 4i\epsilon^{\alpha\beta} \{\psi_{\alpha}, \gamma^{IJ} \psi_{\beta}, \phi^I\}. \quad (3.6)$$

The $\propto \gamma_{\alpha\beta}^{\mu} \gamma_{\dot{A}\dot{B}}^I$ irreducible part of the same relation can be used to obtain the bosonic Chern–Simons equation (3.2), while the $\propto \gamma_{\dot{A}\dot{B}}^{IJK}$ irreducible parts vanish identically.⁸

⁵ I.e., Jacobi identities for the covariant derivatives, $[\mathbb{D}_{\alpha\dot{A}}, \{\mathbb{D}_{\beta\dot{B}}, \mathbb{D}_{\gamma\dot{C}}\}] + (\alpha\dot{A} \begin{smallmatrix} \beta\dot{B} \\ \gamma\dot{C} \end{smallmatrix}) = 0$, etc.

⁶ These equations can be also obtained from consistency of (2.1) with the use of (2.3) (see below).

⁷ On the way of such a derivation of (3.1) one should use the requirement of that the dependence of M_3 coordinates should not be restricted, i.e., no additional conditions on $\partial_i \phi^I$ may occur. Then, coming to the equation $(W_{\dot{A}}^i \dots) \partial_i \phi^I = \kappa_B \gamma_{\dot{A}\dot{B}}^I$, one concludes that $\kappa_B = 0$ and that $W_{\dot{A}}^i = \dots$ where multidots denote the r.h.s. of Eq. (3.1).

⁸ To prove this, one has to use the consequences $\{\phi^L, \phi^I, \{\phi^J, \phi^K, \phi^L\}\} = 0$ and $\epsilon^{IJKLMNPQ} \{\phi^L, \phi^M, \{\phi^N, \phi^P, \phi^Q\}\} = 0$ of the so-called fundamental identity $\{\phi^L, \phi^M, \tilde{e}[\phi^N, \phi^P, \phi^Q]\} = 3[\tilde{e}[\phi^L, \phi^M, \phi^N], \phi^P, \phi^Q]$ (the presence of the density $\tilde{e}(\gamma)$ in the fundamental identity and its absence in its consequences above is not occasional).

² The SO(8) generators acting on **8_s** and **8_c** spinors are $\gamma_{\dot{A}\dot{B}}^{IJ} := (\gamma^{IJ} \tilde{\gamma}^{IJ})_{\dot{A}\dot{B}}$ and $\tilde{\gamma}_{\dot{A}\dot{B}}^{IJ} := (\tilde{\gamma}^{IJ} \gamma^{IJ})_{\dot{A}\dot{B}}$. Among the useful properties of these $d = 8$ γ -matrices are $\gamma_{\dot{A}\dot{B}}^I \gamma_{\dot{B}\dot{C}}^I = \delta_{\dot{A}\dot{B}} \delta_{\dot{B}\dot{C}} + \frac{1}{4} \gamma_{\dot{A}\dot{B}}^{IJ} \tilde{\gamma}_{\dot{B}\dot{C}}^{IJ}$, $\gamma^{IJ} \gamma^{KL} = \gamma^{IJKL} + 4\delta^{[I[K} \gamma^{L]J]} - 2\delta^{[K} \delta^{L]J}$ and $\gamma_{\dot{A}\dot{B}}^{IJKL} \gamma_{\dot{C}\dot{D}}^{IJKL} = 0$.

³ For simplicity, in (2.1) we presented the anticommutator applied to SDiff₃ scalar; the general expression is $\{\mathbb{D}_{\dot{A}}, \mathbb{D}_{\dot{B}}\} = 2i\gamma_{\alpha\beta}^{\mu} \delta_{\dot{A}\dot{B}} \mathcal{D}_{\mu} + 2i\epsilon_{\alpha\beta} W_{\dot{A}\dot{B}}^i$, where $W_{\dot{A}\dot{B}}^i := W_{\dot{A}\dot{B}}^i \partial_i$ and \mathcal{L} is Lie derivative.

⁴ An interesting, although technical, question is whether/how it can be obtained from the consistency of the scalar superfield equation (2.1) and constraints (2.2). However, as far as there is no hope to get an off shell superfield model with 16 supersymmetries, at least in the ‘standard’ superspace, (so that the real question is whether the constraints result in Chern–Simons or in the $D = 3$ SYM equations) in this Letter we impose the super-Chern–Simons equation as a constraint.

To conclude, the superembedding-like equation (2.1), supplemented by the covariant derivative algebra (2.2) with the composite scalar field strength (2.3), restricts field content of the basic octet of $d = 3$, $N = 8$ scalar superfields ϕ^I , depending in addition on three coordinates of a compact space M_3 , to the NB BLG supermultiplet, and, furthermore, accumulates all the equations of motion of the NB BLG model.

4. Conclusions

In this Letter we presented the $N = 8$ superfield description of the Nambu bracket (NB) realization of the Bagger–Lambert–Gustavsson (BLG) model. It is given by an octet of scalar $N = 8$, $d = 3$ superfields ϕ^I which, in addition, depend on the three coordinates y^i of compact space M_3 . This octet of superfields is restricted by Eq. (2.1), which, as we have shown, contains all the equations of motion of the NB BLG model when supplemented (at least, when supplemented) by super-Chern–Simons equation (2.3) (or, equivalently, (2.4)).

We call the basic Eq. (2.1) *superembedding-like equation* because of its relation with the superembedding equation describing one M2-brane in the $d = 3$, $N = 8$ worldvolume superspace which is as follows. To obtain (2.1), one has first to linearize the supermembrane superembedding equation [12], see [13], and to fix the so-called static gauge on the worldvolume superspace, arriving at the equation $D_{\alpha\dot{A}}X^I = 2i\tilde{\gamma}_{\dot{A}B}^I\Psi_{\alpha B}$. Then one replaces the octet of $d = 3$, $N = 8$ superfields $X^I(x, \theta)$ by the octet of superfields depending also on coordinates of M_3 , $X^I(x, \theta) \mapsto \phi^I(x, \theta, y)$ (this automatically produces $\Psi_{\alpha B}(x, \theta) \mapsto \psi_{\alpha B}(x, \theta, y)$) and covariantize the result with respect to the volume preserving diffeomorphisms of M_3 ($D_{\alpha\dot{A}} \mapsto \mathbb{D}_{\alpha\dot{A}} = D_{\alpha\dot{A}} + \zeta_{\alpha\dot{A}}^i \partial_i$).

We hope that our superfield description will be useful in studying the properties of the NB BLG model and in understanding its physical meaning.

Actually, such a way of passing from the complete nonlinear description of one M2-brane in the frame of superembedding approach [12] to the NB BLG model—namely first linearization, and then obtaining nonlinearities by a separate covariantization with respect to SDiff₃—suggests that NB BLG model may be not a description of multiple M2-brane, but rather an independent—and without any doubt very interesting— $d = 3$, $N = 8$ supersymmetric dynamical system.

Actually, a search for alternative candidates on the rôle of multiple M2-brane action can be witnessed. A very incomplete list includes the $N = 6$ supersymmetric model of [14], $d = 3$, $N = 2$ supersymmetric models of [15], as well as very recent construction of a candidate multiple M2-brane bosonic action, similar to the Myers action for the coincident bosonic D-branes, in [16].

The generalization of our on-shell $N = 8$ superstring description for the case of arbitrary 3-algebra seems to be possible⁹ and, in many respects, looks interesting to develop in details.

Note added

When this Letter has been finished, the article [17], addressing the same subject from the pure spinor (pure spinor superspace) perspective, has appeared on the net. A very intriguing statement in [17] is on existence of the superspace action that, if so, would be the first known example of the superfield action with 16 supersymmetries. The action presented in [17] can also be considered as a realization of the harmonic superspace programme of [19] with pure spinors substituting harmonic variables.¹⁰ The approach of [19] overcame some no-go theorems because the number of auxiliary fields in it was infinite. It would be very interesting to analyze the structure of auxiliary field sector in the action of [17].

After the first version of this Letter appeared on the net, certain aspects of measure on the pure spinor space, which were left out and simply assumed to work in [17], were addressed in [20], where a similar pure spinor superspace formulation was also presented for the $N = 6$ model of [14].

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⁹ This is better seen when one writes the SDiff₃ covariant derivatives in terms of Lie brackets of vector fields, $\mathbb{D}\phi^I = D\phi^I + [\zeta, \phi^I]$, $F = ds + \frac{1}{2}[s, s]$. Then these Lie brackets can be substituted by the commutators and the commutators of the field strengths are defined by 3-brackets with scalar and spinor fields, e.g., $[W^{IJ}, \dots] = \{\phi^I, \phi^J, \dots\}$.

¹⁰ As the pure spinors for $D = 10$, $N = 1$ superstring [18] parametrize, modulo overall scale factor, the $SO(10)/U(5)$ coset, one can also state that these pure spinors are harmonic variables.